Reply by Author to B. T. Fang (Regarding his Comment on "A New Method of Solution of the Eigenvalue Problem for Gyroscopic Systems")

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THE matrix product AB of Fang can be identified as the matrix $I^{-1/2}$ of Meirovitch. Then, the matrix P of Fang becomes

$$P = -B^T A^T GAB = -I^{-1/2} GI^{-1/2}$$

because I is symmetric. The eigenvalue problem of Fang, Eq. (3) of the Comment, uses the matrix

$$P^2 = (-I^{-1/2} GI^{-1/2})^2 = I^{-1/2} GI^{-1} GI^{-1/2}$$

and its eigenvalues are λ^2. On the other hand, the eigenvalue problem of Meirovitch, Eqs. (20) and (22) of the Paper, uses the matrix

$$K' = I^{-1/2} KI^{-1/2} = I^{-1/2} G^T I^{-1} GI^{-1/2}$$

and its eigenvalues are ω^2. In Fang's notation, it is clear that

$$K' = P^T P = -P^2$$

because G is skew symmetric. Hence, no material difference exists, except that K' is positive definite if I is positive definite, which in nicer than saying that P^2 is negative definite if I is positive definite. This latter statement, which was not included by Fang in his Comment, is necessary, because only then the added statements concerning the eigenvectors hold true.

One of the powerful aspects of the Paper is the derivation of the real symmetric eigenvalue problem, Eqs. (14) and (17), which permits the solution of the eigenvalue problem for gryoscopic systems by a large variety of existing computer programs. The transformation from Eqs. (14) and (17) to Eqs. (20) and (22) is standard (see, for example, Eq. (4.134) of Ref. 10). Since the book appeared in 1967 and this transformation was known before that date, one may safely assume that there was a computer subroutine available for it. The identification of such a subroutine can be of interest to some readers.

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Index categories: Spacecraft Attitude Dynamics and Control; Structural Dynamic Analysis.
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Comment on "Unsteady Separation Phenomena in a Two-Dimensional Cavity"

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A LOT has been published recently on unsteady separation as desribed in a few very recent review articles.[1-3] Yet it appears that many investigators have misunderstood some of the basic ideas that have been recently advocated by a school of thought to which I belong. O'Brien's paper[4] on "Unsteady Separation Phenomena in a Two-Dimensional Cavity" is a typical example.

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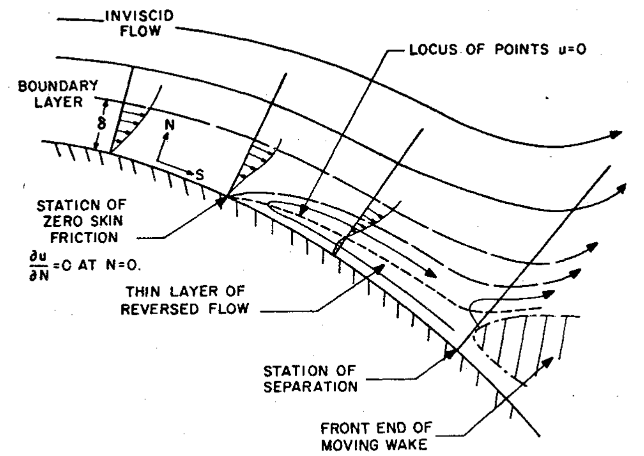

Fig. 1 Schematic sketch of the streamline configuration in the neighborhood of unsteady separation. The dash-dot line denotes the boundary of the wake and does not coincide with any streamline.

I believe that Ref. 4 represents a fine piece of work, and there is nothing wrong either with respect to the mathematical model or with the physical conclusions the author draws for this particular problem. I only disagree with the way she compares qualitatively her results with our work. The physical problem that we have been working on is totally different. We are concerned with external flows of very large Reynolds numbers Re and the well known phenomenon that solutions of the Navier-Stokes equations for $1/\mathrm{Re} \to 0$ and $1/\mathrm{Re} = 0$ are totally different. We define separation as the point where the flow leaves the solid boundaries and turns into the freestream thus generating a wake. A careful study of the steady boundary layer, which in the limit of $\mathrm{Re} \to \infty$ has infinitely small thickness,revealed that at separation the wall shear vanishes[1]. This is not true though for unsteady flow[1] as depicted schematically in Fig. 1 which I borrowed from Ref. 1. The study of unsteady separation is of paramount importance in aerodynamics since it governs phenomena like unsteady airfoil stall, rotating stall of axial flow compressors, flutter, and others.

It may appear that our differences with O'Brien are differences of terminology. She is certainly free to use the term separation for points where streamlines emanate from the wall. I would rather call these points, points of detachment, and I feel that physically they are no different than a point of a rear stagnation. O'Brien though has qualitatively compared in her conclusions her results with our work. The comparison is not very successful. Her phenomena are totally viscous and in her case I believe that the solution for $1/\mathrm{Re} \to 0$ and $1/\mathrm{Re} = 0$ are the same. The catalytic effect of viscosity $\mu \to 0$ is absent. Points of dettachment in unsteady flow of the kind O'Brien has studied are points like the one marked by $\partial u/\partial N = 0$ in Fig. 1 and do not seem to have any major engineering significance.

O'Brien criticizes the use of the boundary-layer equations, and yet I do not know of any other realistic method of calculating external flows for practical engineering problems, especially if the boundary layer is turbulent. Her full momentum equations do not show any singular behavior at "her" point of "separation", a fact which is well known for years[1]. O'Brien then proceeds to claim that her method permits a clearer view of the physical process. I agree that her results are very interesting, but the phenomenon she is trying to illuminate bears no resemblance to unsteady separation.

A minor point perhaps could be added here. With regard to the Moore Rott Sears model of "midstream separation" a lot of analytical and experimental work has been published[1]. A sketch of a "four-way partition" of the flow or equivalently a saddle-point streamline configuration was included in Ref. 5. Trifurcation of the flow is certainly unrealistic, and I know from my discussions with Moore and his co-workers that their sketches simply did not show any details in the wake region. They did not propose a trifurcation configuration.†

References

[1] Sears, W.R. and Telionis, D.P., "Boundary Layers Separation in Unsteady Flow," *SIAM Journal of Applied Mathematics*, Vol. 28, Jan. 1975, pp. 215-235.

[2] Riley, N, "Unsteady Laminar Boundary Layers," *SIAM Review*, Vol. 17, April 1975, pp. 274-297.

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[4] O'Brien, V., "Unsteady Separation Phenomena in a Two-Dimensional Cavity,"*AIAA Journal*, Vol. 13, March 1975, pp. 415-416.

[5] Sears, W.R. and Telionis. D.P., "Unsteady Boundary Layer Separation,"*Recent Research of Unsteady Boundary Layers, Proceedings of a Symposium of the International Union of Theoretical and Applied Mechanics*, Vol. 1, edited by E.A. Eichelbrenner, Quebec, Canada, May 1971, pp. 404-447.

†Note added in proof: Communicating privately with Dr. O'Brien I resolved my misunderstanding with regard to "trifurcation of the flow." By this term we understand a point where three streamlines meet which is not possible unless the two of the streamlines coincide with a solid boundary. Dr. O'Brien on the other hand implies a saddle point streamline configuration where two of the streamlines form a closed loop thus partitioning the space into three aeral domains.

Reply by Author to D. P. Telionis

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MY article[1] presents a particular internal flow problem, but I fully intended that it apply as an example for unsteady separation phenomena from two-dimensional solid bodies in a variety of flow situations. Discussion of unsteady flow procssses,[2] as in many flow processes that are not completely understood, suffers from semantic problems. As used in my article, detachment and reattachment apply to all (instantaneous) streamlines that intersect the solid boundary, whatever the characteristic Reynolds number, internal or external flows. A streamline having both a detachment point and a reattachment point is, per force, a closed recirculation region. In aerodynamic parlance, this is a "separation bubble". It may be thick or thin relative to the upstream boundary layer. If it is thick, it appears the boundary layer is breaking away from the body. Yet true boundary-layer "breakaway"[3] is generally reserved for open separated regions that merge with the wake without reattachment to the body; this is called "separation" in Ref. 2 and sometimes "blow-up" in Ref. 4. Common imprecise use of the term "boundary-layer separation" lumps together thin and thick separation bubbles along with true breakaway as if there were no distinction. In each case, the local flow patterns near detachment and/or reattachment points can only be accurately revealed by regular full Navier-Stokes solutions. The common feature is *coincidence of velocity and vorticity zeroes.* This amounts to the usual "vanishing wall-shear" for steady flow, but the criterion applies as well to unsteady shear flow, though not yet properly accepted. We could also speak of mean detachment points and reattachment ones for pulsatile or turbulent flows.

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Steady boundary-layer theory has been applied successfully to high Reynolds number external flow problems. Yet the distinction between the apparent singularity of the boundary-layer solution and regular behavior at detachment for the full momentum equation[5] is well-known. (The latter is more accurate, of course.) Likewise the use of unsteady boundary-layer equations,[2,4,6] though used for practical engineering estimates for high Reynolds numbers, cannot be accurate for local flow details near detachment. Such analyses are always incomplete in the sense of Riley (Ref. 7 p. 283). The claim in Ref. 2 that a point of detachment (i.e. "my separation") has no major engineering significance is arguable, because it must always precede the bulk flow reversal region (thick or thin, open or closed, transient or permanent. Such flow regions can seriously affect heat or mass transfer.

Finally, if the body surface is moving forward in the frame of reference, the stagnation-separation point (in the Eulerian description) must occur off the body in the freestream. Zero wall-shear (on the body surface) predicts nothing about the streamline intersection. The "saddle-point streamline configuration" (Ref. 4, Fig. 1a) is my four-way partition (Points C & D, Fig. 2, Ref. 1). However, generally the orthogonal intersection need not be parallel-normal with respect to the solid wall. On the other hand, the unsteady shedding of vortices as revealed by full momentum equation calculations[8,9] does not involve such intersections but an osculating streamline ('trifurcation') where the vorticity is not zero. This is clearly a different thing.

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[1] O'Brien, V., "Unsteady Separation Phenomena in a Two-Dimensional Cavity," *AIAA Journal*, Vol. 13, March 1975, pp. 415-416.

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Comment on "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup"

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RECENTLY Rossow[1] introduced two hypothetical vortex wakes and explored, through the use of the discrete-

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